

REAL HYPERSURFACES IN COMPLEX TWO-PLANE GRASSMANNIANS WITH GTW REEB LIE DERIVATIVE STRUCTURE JACOBI OPERATOR

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ABSTRACT. Using generalized Tanaka-Webster connection, we considered a real hypersurface M in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$ when the GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative. Next using the method of simultaneous diagonalization, we prove a complete classification for a real hypersurface in $G_2(\mathbb{C}^{m+2})$ satisfying such a condition. In this case, we have proved that M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$.

INTRODUCTION

For real hypersurfaces with parallel curvature tensor, many differential geometers studied in complex projective spaces or in quaternionic projective spaces ([9, 13, 14]). Different point of view, it is attractive to classify real hypersurfaces in complex two-plane Grassmannians with certain conditions. For example, there is some result about parallel structure Jacobi operator (For more detail, see [7, 8]). It is natural to question about complex two-plane Grassmannians.

As an ambient space, a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$ consists of all complex two-dimensional linear subspaces in \mathbb{C}^{m+2} . This Riemannian symmetric space is the unique compact irreducible Riemannian manifold being equipped with both a Kähler structure J and a quaternionic Kähler structure \mathfrak{J} not containing J . Then, we could naturally consider two geometric conditions for hypersurfaces M in $G_2(\mathbb{C}^{m+2})$, namely, that a 1-dimensional distribution $[\xi] = \text{Span}\{\xi\}$ and a 3-dimensional distribution $\mathfrak{D}^\perp = \text{Span}\{\xi_1, \xi_2, \xi_3\}$ are both invariant under the shape operator A of M ([3]), where the *Reeb* vector field ξ is defined by $\xi = -JN$, N denotes a local unit normal vector field of M in $G_2(\mathbb{C}^{m+2})$ and the *almost contact 3-structure* vector fields $\{\xi_1, \xi_2, \xi_3\}$ are defined by $\xi_\nu = -J_\nu N$ ($\nu = 1, 2, 3$).

By using the result in Alekseevskii [1], Berndt and Suh [3] proved the following result about space of *type* (A)(sentence about (A)) and *type* (B)(one about (B)):

Theorem A. *Let M be a connected orientable real hypersurface in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$. Then both $[\xi]$ and \mathfrak{D}^\perp are invariant under the shape operator of M if and only if*

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- (A) M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$,
or
(B) m is even, say $m = 2n$, and M is an open part of a tube around a totally
geodesic $\mathbb{H}P^n$ in $G_2(\mathbb{C}^{m+2})$.

When we consider the Reeb vector field ξ in the expression of the curvature tensor R for a real hypersurface M in $G_2(\mathbb{C}^{m+2})$, the structure Jacobi operator R_ξ can be defined in such as

$$R_\xi(X) = R(X, \xi)\xi,$$

for any tangent vector field X on M .

Using the structure Jacobi operator R_ξ , Jeong, Pérez and Suh [7] considered a notion of *parallel structure Jacobi operator*, that is, $\nabla_X R_\xi = 0$ for any vector field X on M , and gave a non-existence theorem. And the authors [8] considered the general notion of \mathfrak{D}^\perp -*parallel structure Jacobi operator* defined in such a way that $\nabla_{\xi_i} R_\xi = 0$, $i = 1, 2, 3$, which is weaker than the notion of parallel structure Jacobi operator. They also gave a non-existence theorem.

By the way, the Reeb vector field ξ is said to be *Hopf* if it is invariant under the shape operator A . The one dimensional foliation of M by the integral manifolds of the Reeb vector field ξ is said to be the *Hopf foliation* of M . We say that M is a *Hopf hypersurface* in $G_2(\mathbb{C}^{m+2})$ if and only if the Hopf foliation of M is totally geodesic. By the formulas in Section 1 it can be easily checked that M is Hopf if and only if the Reeb vector field ξ is Hopf.

Now, instead of the Levi-Civita connection for real hypersurfaces in Kähler manifolds, we consider another new connection named *generalized Tanaka-Webster connection* (in short, let us say the *GTW connection*) $\hat{\nabla}^{(k)}$ for a non-zero real number k ([10]). This new connection $\hat{\nabla}^{(k)}$ can be regarded as a natural extension of Tanno's generalized Tanaka-Webster connection $\hat{\nabla}$ for contact metric manifolds. Actually, Tanno [17] introduced the generalized Tanaka-Webster connection $\hat{\nabla}$ for contact Riemannian manifolds by using the canonical connection on a nondegenerate, integrable CR manifold.

On the other hand, the original *Tanaka-Webster connection* ([16, 18]) is given as a unique affine connection on a non-degenerate, pseudo-Hermitian CR manifolds associated with the almost contact structure. In particular, if a real hypersurface in a Kähler manifold satisfies $\phi A + A\phi = 2k\phi$ ($k \neq 0$), then the g-Tanaka-Webster connection $\hat{\nabla}^{(k)}$ coincides with the Tanaka-Webster connection.

Related to GTW connection, due to Jeong, Pak and Suh ([5, 6]), the *GTW Lie derivative* was defined by

$$(1) \quad \hat{\mathcal{L}}_X^{(k)} Y = \hat{\nabla}_X^{(k)} Y - \hat{\nabla}_Y^{(k)} X,$$

where $\hat{\nabla}_X^{(k)} Y = \nabla_X Y + g(\phi AX, Y)\xi - \eta(Y)\phi AX - k\eta(X)\phi Y$, $k \in \mathbb{R} \setminus \{0\}$.

In this paper, using the GTW Lie derivative, we consider a condition that the *GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative*, that is,

$$(2) \quad (\hat{\mathcal{L}}_\xi^{(k)} R_\xi)Y = (\mathcal{L}_\xi R_\xi)Y,$$

for any tangent vector field Y in M . Using above notion, we have a classification theorem as follows:

Main Theorem. *Let M be a connected orientable Hopf hypersurface in a complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$, $m \geq 3$. If the GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative and the Reeb curvature is non-vanishing constant along the Reeb vector field, then M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$.*

As a corollary, we consider a condition stronger than the condition (2) as follows:

$$(\hat{\mathcal{L}}_X^{(k)} R_\xi)Y = (\mathcal{L}_X R_\xi)Y$$

for any tangent vector fields X, Y in M . Then we assert the following

Corollary. *There do not exist any connected orientable Hopf real hypersurfaces in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, with $(\hat{\mathcal{L}}_X^{(k)} R_\xi)Y = (\mathcal{L}_X R_\xi)Y$ when the Reeb curvature is constant along the direction of the Reeb vector field.*

In section 1, we introduce basic equations in relation to the structure Jacobi operator and prove the key lemmas which will be useful to proceed our main theorem. In section 2, we give a complete proof of the main theorem and corollary, respectively. In this paper, we refer to [1, 3, 4, 7, 11] for Riemannian geometric structures of $G_2(\mathbb{C}^{m+2})$ and its geometric quantities, respectively.

1. KEY LEMMAS

In this section, we introduce some fundamental equation of structure Jacobi operator and lemmas.

$$\begin{aligned} R_\xi X &= R(X, \xi)\xi \\ &= X - \eta(X)\xi \\ (1.1) \quad &- \sum_{\nu=1}^3 \left\{ \eta_\nu(X)\xi_\nu - \eta(X)\eta_\nu(\xi)\xi_\nu + 3g(\phi_\nu X, \xi)\phi_\nu \xi + \eta_\nu(\xi)\phi_\nu \phi X \right\} \\ &+ \alpha AX - \alpha^2 \eta(X)\xi, \end{aligned}$$

for any tangent field X on M .

In [5], they defined the GTW Lie derivative as follows:

$$\hat{\mathcal{L}}_X^{(k)} Y = \hat{\nabla}_X^{(k)} Y - \hat{\nabla}_Y^{(k)} X,$$

where $\hat{\nabla}_X^{(k)} Y = \nabla_X Y + F_X Y$, $F_X Y = g(\phi AX, Y)\xi - \eta(Y)\phi AX - k\eta(X)\phi Y$. The operator $F_X Y$ said to be the *generalized Tanaka-Webster operator* (in short, GTW operator). Putting $X = \xi$ and $Y = \xi$, the GTW operator is written as

$$(1.2) \quad F_\xi Y = -k\phi Y \text{ and } F_X \xi = -\phi AX, \text{ respectively.}$$

For an (1-1) type tensor R_ξ , this condition $(\hat{\mathcal{L}}_X^{(k)} R_\xi)Y = (\mathcal{L}_X R_\xi)Y$ is equivalent to

$$(1.3) \quad F_X(R_\xi Y) - F_{R_\xi Y} X - R_\xi F_X Y + R_\xi F_Y X = 0.$$

Replacing $X=\xi$ in (1.3), we get

$$(1.4) \quad -k\phi R_\xi Y + \phi A R_\xi Y + kR_\xi \phi Y - R_\xi \phi AY = 0.$$

Since R_ξ is a symmetric tensor field, taking symmetric part of (1.4), we have

$$(1.5) \quad kR_\xi\phi Y - R_\xi A\phi Y - k\phi R_\xi Y + A\phi R_\xi Y = 0.$$

Subtracting (1.5) from (1.4), we obtain

$$(1.6) \quad (\phi A - A\phi)R_\xi Y = R_\xi(\phi A - A\phi)Y.$$

Therefore, this condition that the GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative has such a geometric condition, that is, $(\phi A - A\phi)$ and R_ξ commute with each other.

Putting $Y = \xi$ in (1.3) and using (1.2), (1.3) is replaced by

$$(1.7) \quad R_\xi(\phi AX) - kR_\xi(\phi X) = 0.$$

Taking the transpose part on (1.7), we get

$$(1.8) \quad -A\phi R_\xi X + k\phi R_\xi X = 0.$$

By using above these equations, we can give two lemmas which contribute to prove our main theorem.

Lemma 1.1. *Let M be a Hopf hypersurface M in $G_2(\mathbb{C}^{m+2})$. If the GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative of this operator and the principal curvature α is constant along the direction of the Reeb vector field ξ , then the Reeb vector field ξ belongs to the distribution \mathfrak{D} or the distribution \mathfrak{D}^\perp .*

Proof. Let us put $\xi = \eta(X_0)X_0 + \eta_1(\xi_1)\xi_1$, for some unit vector fields $X_0 \in \mathfrak{D}$ and $\xi_1 \in \mathfrak{D}^\perp$. If $\alpha = 0$, then $\xi \in \mathfrak{D}$ or $\xi \in \mathfrak{D}^\perp$, which is proved by Pérez and Suh ([15]). So, we consider the other case $\alpha \neq 0$.

Putting $X = \xi_1$ into (1.1) and using $A\xi_1 = \alpha\xi_1$, we have

$$(1.9) \quad R_\xi(\xi_1) = \alpha^2\xi_1 - \alpha^2\eta(\xi_1)\xi.$$

Replacing $X = \phi\xi_1$ into (1.1), (1.1) becomes

$$(1.10) \quad R_\xi(\phi\xi_1) = (\alpha^2 + 8\eta^2(X_0))\phi_1\xi.$$

Putting $X = \xi$ into (1.3) and using (1.2), (1.1) is written as

$$(1.11) \quad -k\phi R_\xi Y + \phi A R_\xi Y + kR_\xi(\phi Y) - R_\xi(\phi AY) = 0.$$

Substituting $Y = \xi_1$ in the above equation and using (1.9), (1.10), it becomes

$$(1.12) \quad 8(k - \alpha)\eta^2(X_0)\phi_1\xi = 0.$$

Taking the inner product with $\phi_1\xi$, we get

$$(1.13) \quad 8(k - \alpha)\eta^4(X_0) = 0.$$

This equation induces that $k = \alpha$ or $\eta^4(X_0) = 0$. Therefore, it completes the proof of our Lemma. \square

In next section, we will give a complete proof of our main theorem. In order to do this, first we consider the case that $\xi \in \mathfrak{D}^\perp$. Without loss of generosity, we may put $\xi = \xi_1$.

Lemma 1.2. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$ when the Reeb curvature is non-vanishing. If the GTW Reeb Lie derivative of the structure Jacobi operator coincides with the Reeb Lie derivative of this operator and the Reeb vector field ξ is belong to the distribution \mathfrak{D}^\perp , then the shape operator A commutes with the structure tensor ϕ .*

Proof. Putting $\xi = \xi_1$ in (1.1), we get

$$(1.14) \quad R_\xi X = X - \eta(X)\xi - \phi_1\phi X + \alpha AX - \alpha^2\eta(X)\xi + 2\eta_2(X)\xi_2 + 2\eta_3(X)A\xi_3.$$

Replacing X with AX in (1.14), it is written as

$$(1.15) \quad R_\xi AX = AX - \alpha\eta(X)\xi - \phi_1\phi AX + \alpha A^2X - \alpha^3\eta(X)\xi + 2\eta_2(AX)\xi_2 + 2\eta_3(AX)A\xi_3.$$

And applying the shape operator A on (1.14), (1.14) becomes

$$(1.16) \quad AR_\xi X = AX - \alpha\eta(X)\xi - A\phi_1\phi X + \alpha A^2X - \alpha^3\eta(X)\xi + 2\eta_2(X)A\xi_2 + 2\eta_3(X)A\xi_3.$$

On the other hand, applying the structure tensor field ϕ to the equation (1.8) in [12], we get

$$(1.17) \quad AX = \alpha\eta(X)\xi + 2\eta_2(AX)\xi_2 + 2\eta_3(AX)\xi_3 - \phi\phi_1AX.$$

Taking the symmetric part of (1.17), we obtain

$$(1.18) \quad AX = \alpha\eta(X)\xi + 2\eta_2(X)A\xi_2 + 2\eta_3(X)A\xi_3 - A\phi_1\phi X.$$

Putting $\nu = 1$ in the first equation of (1.5) in [5], it becomes

$$(1.19) \quad \phi\phi_1X = \phi_1\phi X.$$

Using (1.17), (1.18) and subtracting (1.16) from (1.15), we have

$$(1.20) \quad R_\xi AX = AR_\xi X.$$

From (1.20) and putting $Y = X$, (1.8) is written as

$$(1.21) \quad A(R_\xi\phi - \phi R_\xi)X = (R_\xi\phi - \phi R_\xi)AX.$$

Putting $X = \phi X$ in (1.14), we have

$$(1.22) \quad R_\xi\phi X = \phi X - \phi_1\phi^2X + \alpha A\phi X + 2\eta_2(\phi X)\xi_2 + 2\eta_3(\phi X)A\xi_3.$$

Applying the structure tensor field ϕ to (1.14), we get

$$(1.23) \quad \phi R_\xi X = \phi X - \phi\phi_1\phi X + \alpha\phi AX + 2\eta_2(X)\phi\xi_2 + 2\eta_3(X)A\phi\xi_3.$$

Subtracting (1.23) from (1.22), we obtain

$$(1.24) \quad (R_\xi\phi - \phi R_\xi)X = \alpha(A\phi - \phi A)X.$$

Using the equation (1.24), the equivalent condition of (1.21) is this one as

$$(1.25) \quad \alpha A(A\phi - \phi A)X = \alpha(A\phi - \phi A)AX.$$

By our assumption $\alpha \neq 0$, the above equation can be replaced by

$$(1.26) \quad A(A\phi - \phi A)X = (A\phi - \phi A)AX.$$

Because of (1.26), there is a common basis $\{e_i \mid i = 1, \dots, 4m-1\}$ such that

$$(1.27) \quad Ae_i = \lambda_i e_i$$

and

$$(1.28) \quad (A\phi - \phi A)e_i = \gamma_i e_i.$$

Using (1.27), (1.28) becomes

$$(1.29) \quad \gamma_i e_i = A\phi e_i - \phi A e_i = A\phi e_i - \lambda_i \phi e_i.$$

Taking the inner product with e_i , we get $\gamma_i = 0$.

Since the eigenvalue γ_i vanishes for all i , from (1.28) we conclude that

$$(1.30) \quad A\phi - \phi A = 0.$$

Consequently, we proved this lemma. \square

2. PROOF OF THE MAIN THEOREM

Let us consider a Hopf hypersurface M in $G_2(\mathbb{C}^{m+2})$ with $(\hat{\mathcal{L}}_\xi^{(k)} R_\xi)Y = (\mathcal{L}_\xi R_\xi)Y$. By Lemma 1 in section 1, we can conclude that the Reeb vector field ξ in M belongs either to the distribution \mathfrak{D} or \mathfrak{D}^\perp .

Then, we can divide the following two cases:

- Case I: $\xi \in \mathfrak{D}^\perp$
- Case II: $\xi \in \mathfrak{D}$

Now, we check the first case in our consideration.

If $\xi \in \mathfrak{D}^\perp$, by Theorem A and Lemma 2, we can assert that M is locally congruent to the model space of type (A). We have to check if the model space of type (A) satisfies the condition $(\hat{\mathcal{L}}_\xi^{(k)} R_\xi)Y = (\mathcal{L}_\xi R_\xi)Y$ or not. For type (A)-space, detail information (eigenspaces, corresponding eigenvalues, and multiplicities) was given in [3].

Putting $X = \xi$ in (1.3), we get the equivalent condition of $(\hat{\mathcal{L}}_\xi^{(k)} R_\xi)Y = (\mathcal{L}_\xi R_\xi)Y$ as follows:

$$(2.1) \quad -k\phi R_\xi Y + \phi A R_\xi Y + kR_\xi \phi Y - R_\xi \phi A Y = 0.$$

On the other hand, putting $\xi = \xi_1$ into (1.1), we get

$$(2.2) \quad R_\xi X = X - \eta(X)\xi - \phi_1 \phi X + \alpha A X - \alpha^2 \eta(X)\xi + 2\eta_2(X)\xi_2 + 2\eta_3(X)\xi_3.$$

Using (2.1) and (2.2), we get the following result:

$$(2.3) \quad -k\phi(R_\xi Y) + \phi A(R_\xi Y) + R_\xi k\phi Y - R_\xi \phi A Y = \begin{cases} 0, & \text{if } Y \in T_\alpha \\ 0, & \text{if } Y \in T_\beta \\ 0, & \text{if } Y \in T_\lambda \\ 0, & \text{if } Y \in T_\mu. \end{cases}$$

Therefore, we can assert that if $\xi \in \mathfrak{D}^\perp$, then M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$.

If the Reeb vector field $\xi \in \mathfrak{D}$, due to [11], we can assert that M is locally congruent to space of type (B). It remains whether type (B)-space satisfies this condition $(\hat{\mathcal{L}}_X^{(k)} R_\xi)Y = (\mathcal{L}_X R_\xi)Y$. Also, by using information of type (B)-space given in [3], we can check this problem.

We suppose that type (B)-space satisfies $(\hat{\mathcal{L}}_\xi^{(k)} R_\xi)Y = (\mathcal{L}_\xi R_\xi)Y$. Then, as an equivalent condition, this space must satisfy

$$(2.4) \quad -k\phi(R_\xi Y) + \phi A(R_\xi Y) + R_\xi k\phi Y - R_\xi \phi A Y = 0.$$

Since ξ is belong to \mathfrak{D} , the structure Jacobi operator in $G_2(\mathbb{C}^{m+2})$ can be replaced as follows:

$$(2.5) \quad R_\xi X = X - \eta(X)\xi - \sum_{\nu=1}^3 \left\{ \eta_\nu(X)\xi_\nu + 3g(\phi_\nu X, \xi)\phi_\nu \xi \right\} + \alpha AX - \alpha^2 \eta(X)\xi.$$

Applying $Y = \phi_1 \xi \in T_\gamma$ into (2.4) and using (2.5), we get

$$(2.6) \quad k(4 - \alpha\beta)\xi_1 = 0.$$

Since $k \neq 0$ and $\alpha\beta = 4$, this makes a contradiction.

Hence summing up these assertions, we have given a complete proof of our main theorem in the introduction. \square

3. PROOF OF COROLLARY

In this section, we consider another problem for this condition

$$(3.1) \quad (\hat{\mathcal{L}}_X^{(k)} R_\xi)Y = (\mathcal{L}_X R_\xi)Y,$$

for any tangent vector fields X, Y in M .

If the Reeb curvature is non-vanishing, the condition $\phi A = A\phi$ have already proved in Lemma 1.2. Thus, we now consider only the case that α is vanishing. Under these assumptions, we give the following lemma.

Lemma 3.1. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$ with vanishing the Reeb curvature. If the GTW Reeb Lie derivative of structure Jacobi operator coincides with Reeb Lie derivative of this operator and the Reeb vector field ξ is belong to the distribution \mathfrak{D}^\perp , then shape operator A and the structure tensor ϕ commute each other.*

Proof. Recall that (1.3) was given by

$$(3.2) \quad F_X(R_\xi Y) - F_{R_\xi Y}X - R_\xi F_X Y + R_\xi F_Y X = 0.$$

Putting $X = \xi$ in the above equation and using (1.7), (1.8), (3.2) is written as

$$(3.3) \quad (\phi A - A\phi)R_\xi Y = 0.$$

Applying $\alpha = 0$ in (2.2), it becomes

$$(3.4) \quad R_\xi X = X - \eta(X)\xi - \phi_1 \phi X + 2\eta_2(X)\xi_2 + 2\eta_3(X)A\xi_3.$$

On the other hand, applying ϕ and $X = \phi X$ to (1.18), respectively, we have

$$(3.5) \quad \begin{aligned} \phi AX &= 2\eta_2(X)\phi A\xi_2 + 2\eta_3(X)\phi A\xi_3 - \phi A\phi_1 \phi X, \\ A\phi X &= 2\eta_3(X)A\xi_2 - 2\eta_2(X)A\xi_3 - A\phi_1 \phi^2 X. \end{aligned}$$

Combining (3.3), (3.4), (3.5) and using (1.19), we get

$$(3.6) \quad 2(\phi A - A\phi)Y = 0.$$

Therefore we also get the same conclusion in case of $\alpha = 0$. \square

By Lemmas 1.2 and 3.1, we can assert that if $\xi \in \mathfrak{D}^\perp$, then M is the model space of type (A). Now we need to check if the space of type (A) satisfies (3.1) or not.

Then the type (A)-space must satisfy the following condition

$$(3.7) \quad F_X R_\xi Y - F_{R_\xi Y}X - R_\xi F_X Y + R_\xi F_Y X = 0.$$

Putting $Y = \xi$ into (3.7), we have

$$(3.8) \quad R_\xi \phi AX - kR_\xi \phi X = 0.$$

By using (3.4), (3.8) becomes

$$(3.9) \quad \begin{aligned} & \phi AX + \phi_1 AX + \alpha A\phi AX + 2\eta_3(AX)\xi_2 - 2\eta_2(AX)\xi_3 \\ & - k\phi X - k\phi_1 X - k\alpha A\phi X - 2k\eta_3(X)\xi_2 + 2k\eta_2(X)\xi_3 = 0. \end{aligned}$$

Replacing ξ_2 into X, we get

$$(3.10) \quad (\alpha\beta + 2)(k - \beta)\xi_3 = 0.$$

Taking the inner product with ξ_3 , the above equation implies $\alpha\beta = -2$ or $k = \beta$. However, since $k \neq 0$, $\alpha = \sqrt{8} \cot(\sqrt{8}r)$ and $\beta = \sqrt{2} \cot(\sqrt{2}r)$, this makes a contradiction.

Hence we can assert our corollary in the introduction. \square

REFERENCES

- [1] D.V. Alekseevskii, *Compact quaternion spaces*, Funct. Anal. Appl., 1968, **2**, 11-20.
- [2] J. Berndt, *Riemannian geometry of complex two-plane Grassmannian*, Rend. Sem. Mat. Univ. Politec. Torino 1997, **55**, 19–83.
- [3] J. Berndt and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians*, Monatsh. Math., 1999, **127**, 1–14.
- [4] J. Berndt and Y.J. Suh, *Isometric flows on real hypersurfaces in complex two-plane Grassmannians*, Monatsh. Math., 2002, **137**, 87–98.
- [5] I. Jeong, E. Pak, and Y. J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with generalized Tanaka-Webster invariant shape operator*, J. Math. Physics, Analysis, Geometry, 2013, **9**, 360–378
- [6] I. Jeong, E. Pak, and Y. J. Suh, *Lie Invariant Shape Operator for Real Hypersurfaces in Complex Two-Plane Grassmannians*, J. Math. Physics, Analysis, Geometry, 2013, **9**, 455–475
- [7] I. Jeong, J. D. Pérez and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with parallel structure Jacobi operator*, Acta Math. Hungar., 2009, **122**, 173–186.
- [8] I. Jeong, C. J. G. Machado, J. D. Pérez and Y.J. Suh, *Real hypersurfaces in complex two-plane Grassmannians with \mathfrak{D}^\perp -parallel structure Jacobi operator*, Internat. J. Math., 2011, **22**, 655–673.
- [9] U-H. Ki, J. D. Pérez, F. G. Santos and Y.J. Suh, *Real hypersurfaces in complex space forms with ξ -parallel Ricci tensor and structure Jacobi operator*, J. Korean Math. Soc., 2007, **44**, 307–326.
- [10] M. Kon, *Real hypersurfaces in complex space forms and the generalized-Tanaka-Webster connection*, Proceeding of the 13th International Workshop on Differential Geometry and Related Fields (5–7 Nov. 2009 Daegu, Republic of Korea), National Institute of Mathematical Sciences, 2009, 145–159.
- [11] H. Lee, and Y.J. Suh, *Real hypersurfaces of type B in complex two-plane Grassmannians related to the Reeb vector*, Bull. Korean Math. Soc. **47** (2010), no. 3, 551–561.
- [12] H. Lee, Y.J. Suh, and C.H. Woo, *Real hypersurfaces in complex two-plane Grassmannians with commuting Jacobi operators*, Houston J. Math. **40** (2014), no. 3, 751–766.
- [13] J. D. Pérez, F. G. Santos and Y.J. Suh, *Real hypersurfaces in complex projective space whose structure Jacobi operator is \mathfrak{D} -parallel*, Bull. Belg. Math. Soc. Simon Stevin 2006, **13**, 459–469.
- [14] J. D. Pérez and Y.J. Suh, *Real hypersurfaces of quaternionic projective space satisfying $\nabla_{U_i} R = 0$* , Differential Geom. Appl., 1997, **7**, 211–217.
- [15] J.D. Pérez and Y.J. Suh, *The Ricci tensor of real hypersurfaces in complex two-plane Grassmannians*, J. Korean Math. Soc. **44** (2007), 211–235.
- [16] N. Tanaka, *On non-degenerate real hypersurfaces, graded Lie algebras and Cartan connections*, Japan. J. Math., 1976, **20**, 131–190.

- [17] S. Tanno, *Variational problems on contact Riemannian manifolds*, Trans. Amer. Math. Soc., 1989, **314**, 349–379.
- [18] S.M. Webster, *Pseudo-Hermitian structures on a real hypersurface*, J. Differential Geom., 1978, **13**, 25–41.

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